

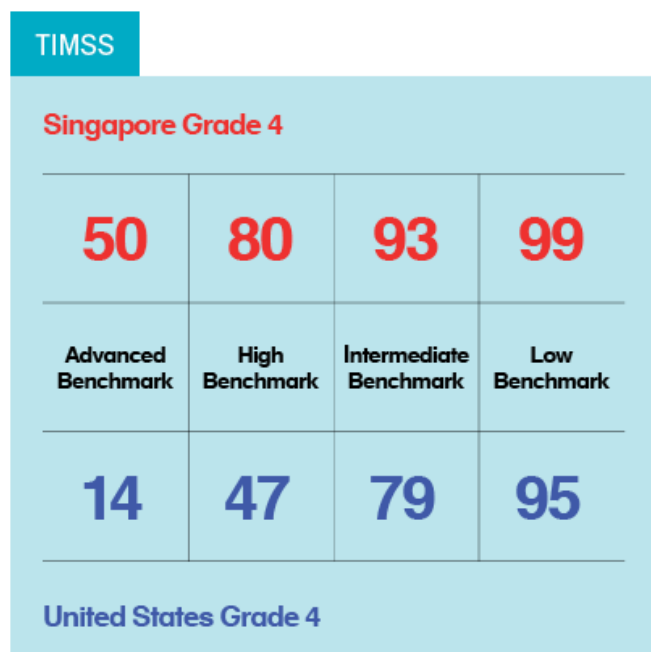
## Why Singapore Math?

### Proven Results

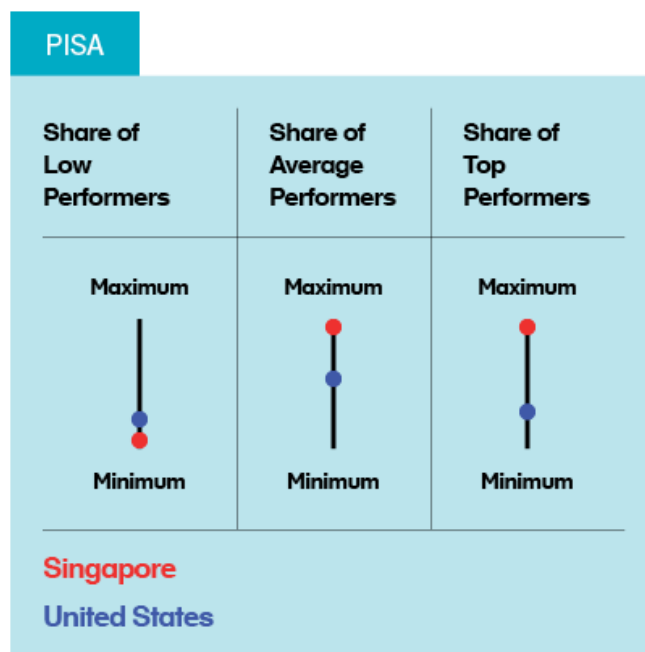
The Singapore Math® method is a highly effective teaching approach based on research of math mastery in Singapore, which consistently ranks at the top in international math testing. The intentional progression of concepts in the Singapore math approach instills a deep understanding of mathematics. An attitude that math is important and approachable is also essential. Students perform at a higher level when their potential for understanding and success is assumed.

### Evidence-based

Two international tests, the TIMSS (Trends in International Mathematics and Science Study) and the PISA (Programme for International Student Assessment), assess math and science competency in countries around the world. Singapore students consistently rank among the top on both tests. Our Singapore math curricula aims to raise U.S. student performance internationally and at home on standardized and state assessments.



*TIMSS 2015 International Results in Mathematics*



*OECD PISA Global Education Survey 2015*

## The Singapore Math Approach

Some of the key features of the Singapore Math method include the CPA (Concrete Pictorial Abstract) approach, number bonds, bar modeling, and mental math. Students learn to think mathematically rather than rely on rote memorization

## Concrete, Pictorial, Abstract (CPA) Approach

The Concrete, Pictorial, Abstract (CPA) approach develops a deep understanding of math through building on existing understanding. This highly effective framework introduces concepts in a tangible way and progresses to increasing levels of abstraction. In the concrete phase, students interact with physical objects to model problems. In the pictorial phase, they make a mental connection between the objects they just handled and visual representations of those objects. For example, real oranges (or counters standing in for oranges) are now represented as drawings of oranges. In the abstract phase, students use symbolic modeling of problems using numbers and math symbols (+, −, ×, ÷).

By varying the methods and phases of CPA fluidly, educators help reinforce important connections. Students work towards math mastery when they view concepts with increasing levels of abstraction over time. Not all lessons include all three CPA stages as application of this approach varies by topic. Instead, CPA principles are woven throughout the curriculum, and support other important strategies such as number bonds, bar modeling, and mental math.

## Number Bonds

Number bonds are a pictorial technique that show the part-whole relationship between numbers. Initially, the whole number is written in one circle, and the parts of the number are written in adjoining circles connected by lines to the first circle. This method helps early elementary students work towards addition and subtraction, and illustrates strategies to solve expressions mentally. Using number bonds fosters a solid number sense that serves students throughout their math education.

This problem from Dimensions Math Textbook 1A shows the stages of the CPA approach, incorporating the use of number bonds to illustrate addition.

5 birds are on a branch.  
2 more birds come.  
How many birds will be on the branch altogether?

### Concrete Stage

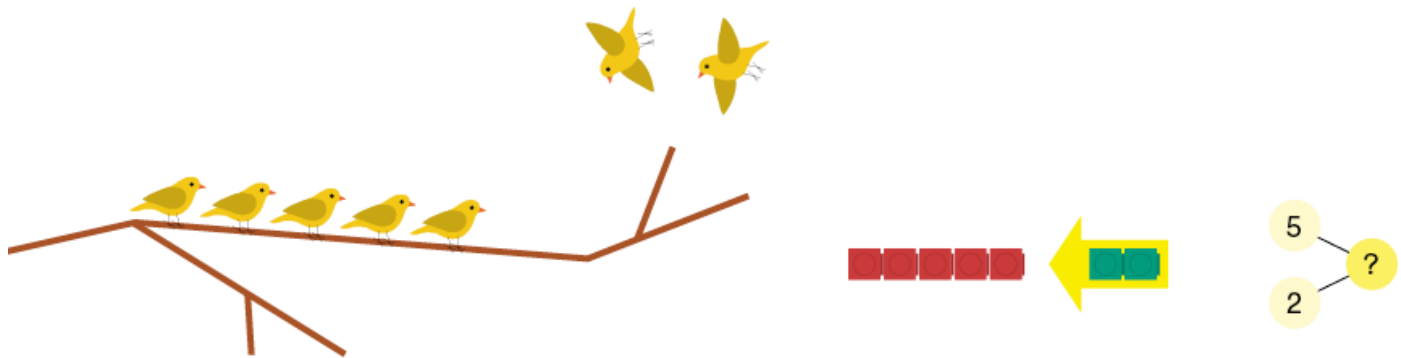
In this stage, teachers lead a classroom activity.

Students represent birds. Have 5 students go to the front of the class and ask the rest of the class how many students there are. Send 2 more students up to the front and ask, “How many students are there now?” Ask students to explain what happened. The interaction introduces students to the problem in a tangible way.

*(Adapted from Dimensions Math Teacher’s Guide 1A)*

### Pictorial Stage

Students are then shown a visual representation of the problem.



*From Dimensions Math Textbook 1A*

### Abstract Stage

Students are then shown an equation of the problem.

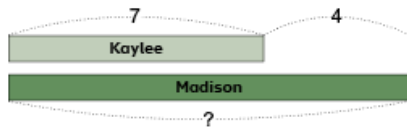
$$5 + 2 = \square$$

## Bar Modeling

Bar models are a versatile and transferable tool that students can use to visualize a range of math concepts, such as fractions, ratios, percentages, and more. Drawing bar models for word problems allows students to determine the knowns and unknowns in a given situation. It extends the CPA approach, especially the pictorial phase, as it allows students to illustrate the mathematical information given in problems. It prepares them to understand more complex math on a conceptual level. This method is most effective when used frequently throughout the program.

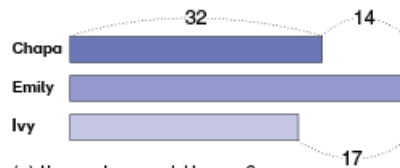
The problems below demonstrate various examples of bar modeling.

Kaylee is 7 years old.  
 Her sister Madison is 4 years older than Kaylee.  
 How old is Madison?



From Dimensions Math Textbook 2A

Chapa saved \$32.  
 Emily saved \$14 more than Chapa.  
 Ivy saved \$17 less than Emily.



- (a) How much money did Ivy save?  
 (b) How much money did the three girls save altogether?

From Dimensions Math Textbook 3A

A set of brushes costs \$5.  
 An acrylic paint set costs 5 times as much as the brushes.  
 How much more does the paint set cost than the brushes?



1 unit  $\rightarrow$  5  
 4 units  $\rightarrow$  4  $\bigcirc$  =

The paint set costs \$ more than the brushes.

From Dimensions Math Textbook 3A

## Mental Math

The Singapore Math approach teaches techniques and skills to easily and accurately perform mental math. These strategies help students develop number sense and flexibility in thinking about numbers. Many mental math strategies involve decomposing numbers into parts, then performing operations on them in a different order from the original expression. The thought processes involved in mental math are often illustrated by number bonds.

Some mental math strategies are taught as early as grade 1. As students progress, they learn to apply new mental math strategies to specific types of problems and adapt ones they already know. Students are encouraged to develop their own strategies, and to use their discernment in deciding when and where to use them.

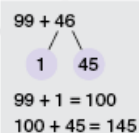
Here are some examples of mental math.

Add 75 and 80.



What other ways could we split the numbers to add?

From Dimensions Math Textbook 2B

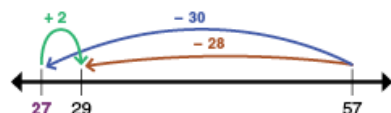


46 + 99  
99 is 1 less than 100.  
46 + 100 = 146  
146 - 1 = 145  
Adding 100 and subtracting 1 is the same as adding 99.



From Dimensions Math Textbook 2B

30 is 2 more than 28.  
 $57 - 30 = 27$   
 $27 + 2 = 29$   
 $57 \xrightarrow{-30} 27 \xrightarrow{+2} 29$



$57 - 28 =$   He has  tickets left.

From Dimensions Math Textbook 3A

$$8 \times 2 = 16$$



$$8 \times 2 = 16$$

$$8 \times 4 =$$



If I know  $8 \times 4$ , I also know  $4 \times 8$ .

$$8 \times 4 = 16 + 16$$



From Dimensions Math Textbook 3A

$$10 \times 4 =$$

$$1 \times 4 =$$

$$9 \times 4 =$$

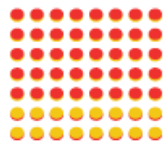
$$4 \times 9 =$$



$$9 \times 4 = 40 - ?$$



From Dimensions Math Textbook 3A



$$5 \times 8 = 40$$

$$2 \times 8 =$$

$$7 \times 8 =$$

$$7 \times 8 = 40 + ?$$



From Dimensions Math Textbook 3B

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